# Deconfinement at finite chemical potential

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**Abstract**

In a confining, renormalisable, Dyson-Schwinger equation model of two-flavour QCD we explore the chemical-potential dependence of the dressed-quark propagator, which provides a means of determining the behaviour of the chiral and deconfinement order parameters, and low-energy pion observables. We find coincident, first order deconfinement and chiral symmetry restoration transitions at $\mu_c = 375 \, \text{MeV}$. $f_\pi$ is insensitive to $\mu$ until $\mu \approx \mu_0 \doteq 0.7 \, \mu_c$ when it begins to increase rapidly. $m_\pi$ is weakly dependent on $\mu$, decreasing slowly with $\mu$ and reaching a minimum 6% less than its $\mu = 0$ value at $\mu = \mu_0$. In a two-flavour free-quark gas at $\mu = \mu_c$ the baryon number density would be approximately $3 \, \rho_0$, where $\rho_0 = 0.16 \, \text{fm}^{-3}$; while in such a gas at $\mu_0$ the density is $\rho_0$.

*Key words:* Field theory at finite chemical potential; Pion properties; Confinement; Dynamical chiral symmetry breaking; Dyson-Schwinger equations

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**1. Introduction.** The production of a quark-gluon plasma and the quantitative exploration of its properties is a primary goal of experimental studies of high energy heavy ion collisions. Present efforts explore the domain of nonzero baryon number density [1]. The theoretical study of finite baryon density in QCD begins with the inclusion of a chemical potential, $\mu$, which modifies the fermion piece of the Euclidean action: $\gamma \cdot \partial + m \to \gamma \cdot \partial - \gamma_4 \mu + m$. Whether conclusions reached via this application of equilibrium statistical field theory can actually be explored in heavy ion collisions is an unresolved question.

With the inclusion of $\mu$ the fermion determinant acquires an explicit imaginary part, in addition to those terms associated with axial anomalies. The $\mu \neq 0$ QCD action being complex entails that the study of finite density is significantly more difficult than that of finite temperature, $T$. As an illustration of this, whereas numerical simulations of finite-$T$ lattice-QCD have been quite extensive [2], requiring only asymmetric lattices, there is currently no numerical simulation algorithm for finite density lattice-QCD. Contemporary studies using the quenched approximation; i.e., eliminating the fermion determinant, encounter a forbidden region, which begins at $\mu = m_\pi/2$ [3]. Since $m_\pi \rightarrow 0$ in the chiral limit this is a serious limitation, preventing a reliable study of chiral symmetry restoration, for example.

At $T = 0 = \mu$ the Dyson-Schwinger equations [DSEs] have been employed extensively in the study of confinement and dynamical chiral symmetry breaking [4]; and the calculation of hadron observables [5]. As the quark-gluon plasma is characterised by deconfinement and chiral symmetry restoration, they also provide a continuum tool for the exploration of the onset and properties of this phase of QCD. The DSEs are a system of coupled integral equations whose solutions, the $n$-point Schwinger functions, are the fully-dressed Euclidean propagators and vertices for the theory. Once all the Schwinger functions are known then the theory is solved. To arrive at a tractable problem one must truncate the system at a given level. Truncations that preserve the global symmetries of a field theory are easy to implement [6]. Preserving the gauge symmetry is more difficult but progress is being made [7].

An often-used approach is to focus on the DSE for the dressed-quark propagator, $S(p)$: the 2-point quark Schwinger function, whose kernel is constructed from the dressed-gluon propagator, $D_{\mu\nu}(k)$, and the dressed-quark-gluon vertex, $\Gamma_\mu(k,p)$. Choosing Ansätze for $D_{\mu\nu}(k)$ and $\Gamma_\mu(k,p)$ one obtains a single integral equation whose solution provides information about quark confinement and dynamical chiral symmetry breaking (DCSB). When the Ansätze are based on sound premises, such as the extensive body of DSE studies relating to the gluon propagator [4,8] and quark-gluon vertex [9], and the results are insensitive to qualitatively equivalent variations of these Ansätze, then the conclusions of such a study can be judged robust.

This approach has been used in a study of deconfinement and chiral symmetry restoration in 2-flavour QCD at finite-$T$ [10]. Therein the quark DSE was solved using the one-parameter model dressed-gluon propagator of Ref. [11], which provided a good description of $\pi$ and $\rho$-meson observables at $T = 0 = \mu$, and a continuum order parameter for deconfinement was introduced. That study established the existence of coincident, second-order deconfinement and chiral symmetry restoration transitions at $T = T_c \approx 150\,\mathrm{MeV}$ with a critical exponent $\beta = 0.33 \pm 0.03$, which is consistent with that of the $N = 4$ Heisenberg magnet: $\beta_H = 0.38 \pm 0.01$. This has been argued [12] to characterise the

universality class containing 2-flavour QCD. Both the pion mass, $m_\pi$, and the pion leptonic decay constant, $f_\pi$, were insensitive to $T$ until $T \approx 0.7 T_c$. However, as $T \to T_c$, the pion mass increased substantially, as thermal fluctuations overwhelmed quark-antiquark attraction in the pseudoscalar channel, until, at $T = T_c$, $f_\pi \to 0$ and there was no bound state. These results confirm those of contemporary numerical simulations of finite-$T$ lattice-QCD [2,13]; and make interesting the exploration of this model at finite chemical potential.

**2. DSE-model of two-flavour QCD.** In a Euclidean space formulation, with $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$ and $\gamma_\mu^\dagger = \gamma_\mu$, the renormalised dressed-quark propagator at $\mu \neq 0$ takes the form:

$$S(\tilde{p}) \doteq -i\vec{\gamma} \cdot \vec{p}\,\sigma_A(\tilde{p}) - i\gamma_4\,\omega_p\,\sigma_C(\tilde{p}) + \sigma_B(\tilde{p})\,, \tag{1}$$

where $\tilde{p} \doteq (\vec{p}, \omega_p \doteq p_4 + i\mu)$, and satisfies the DSE:

$$S(\tilde{p})^{-1} \doteq i\vec{\gamma} \cdot \vec{p}\,A(\tilde{p}) + i\gamma_4\omega_p C(\tilde{p}) + B(\tilde{p}) \tag{2}$$

$$= Z_2^A i\vec{\gamma} \cdot \vec{p} + Z_2(i\gamma_4\omega_p + m_{\mathrm{bm}}) + \Sigma'(\tilde{p})\,. \tag{3}$$

Here $m_{\mathrm{bm}}$ is the Lagrangian current-quark bare mass and the regularised self energy is

$$\Sigma'(\tilde{p}) = i\vec{\gamma} \cdot \vec{p}\,\Sigma'_A(\tilde{p}) + i\gamma_4\omega_p\,\Sigma'_C(\tilde{p}) + \Sigma'_B(\tilde{p})\,; \tag{4}$$

$$\Sigma'_{\mathcal{F}}(\tilde{p}) = \int_q^\Lambda \tfrac{4}{3}g^2 D_{\mu\nu}(\tilde{p} - \tilde{q})\,\tfrac{1}{4}\mathrm{tr}\left[\mathcal{P}_{\mathcal{F}}\gamma_\mu S(\tilde{q})\Gamma_\nu(\tilde{q},\tilde{p})\right]\,, \tag{5}$$

where: $\mathcal{F} = A,\,B,\,C$; $\mathcal{P}_A \doteq -(Z_1^A i\vec{\gamma} \cdot \vec{p}/|\vec{p}|^2)$, $\mathcal{P}_B \doteq Z_1$, $\mathcal{P}_C \doteq -(Z_1 i\gamma_4/\omega_p)$; and $\int_q^\Lambda \doteq \int^\Lambda d^4q/(2\pi)^4$ represents mnemonically a translationally invariant regularisation of the integral, with $\Lambda$ the regularisation mass-scale. Although not explicitly indicated, the solutions, $\sigma_{\mathcal{F}}$, are functions only of $|\vec{p}|^2$ and $\omega_p^2$.

In renormalising we require that

$$S(\vec{p}, \omega_p)^{-1}\Big|_{|\vec{p}|^2 + p_4^2 = \zeta^2}^{\mu=0} = i\vec{\gamma} \cdot \vec{p} + i\gamma_4 p_4 + m_R(\zeta)\,, \tag{6}$$

where $\zeta$ is the renormalisation point and $m_R(\zeta)$ is the renormalised current-quark mass. This entails that the renormalisation constants are:

$$Z_2^A(\zeta^2, \Lambda^2) = 1 - \Sigma'_A(\vec{p}, p_4)\big|_{|\vec{p}|^2 + p_4^2 = \zeta^2}^{\mu=0}\,, \tag{7}$$

$$Z_2(\zeta^2, \Lambda^2) = 1 - \Sigma'_C(\vec{p}, p_4)\big|_{|\vec{p}|^2 + p_4^2 = \zeta^2}^{\mu=0}\,, \tag{8}$$

$$m_R(\zeta^2) = Z_2 m_{\rm bm} + \Sigma'_B(\vec{p}, p_4)|_{|\vec{p}|^2 + p_4^2 = \zeta^2}^{\mu=0} ,\tag{9}$$

and yields the renormalised self energies:

$$\mathcal{F}(\vec{p}, \omega_p) = \xi_{\mathcal{F}} + \Sigma'_{\mathcal{F}}(\vec{p}, \omega_p) - \Sigma'_{\mathcal{F}}(\vec{p}, p_4)|_{|\vec{p}|^2 + p_4^2 = \zeta^2}^{\mu=0} ,\tag{10}$$

where $\mathcal{F} = A$, $B$, $C$; $\xi_A = 1 = \xi_C$ and $\xi_B = m_R(\zeta^2)$.

In studying confinement the $\tilde{p}$-dependence of $A$ and $C$ is qualitatively important since it can conspire with that of $B$ to eliminate free-particle poles in the dressed-quark propagator [14]. Furthermore, in the study of Ref. [15], the $\tilde{p}$-dependence of $A$ and $C$ was a crucial factor in determining the behaviour of bulk thermodynamic quantities such as the pressure and entropy; being responsible for these quantities reaching their respective Stefan-Boltzmann limits only for very large values of $T$ and $\mu$. It is therefore important in any DSE study to retain $A(\tilde{p})$ and $C(\tilde{p})$, and their dependence on $\tilde{p}$.

The DSE-model is specified by the Landau-gauge choice [11]

$$g^2 D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \frac{\mathcal{G}(k^2)}{k^2} ;\tag{11}$$

$$\frac{\mathcal{G}(k^2)}{k^2} = 4\pi^2 d \left[4\pi^2 m_t^2 \delta^4(k) + \frac{1 - {\rm e}^{-[k^2/(4m_t^2)]}}{k^2}\right] .\tag{12}$$

The first term in (12) is an integrable, infrared singularity that provides long-range effects associated with confinement [8]. The second term ensures that, neglecting logarithmic corrections, the propagator has the correct perturbative behaviour at large spacelike-$k^2$. Requiring that the large-distance effects associated with $\delta^4(k)$ are completely cancelled at small-distances by the second term fixes the ratio of the coefficients of these two terms in (12). Since $\mathcal{G}(k^2)/k^2$ doesn't have a Lehmann representation it can be interpreted as describing a confined gluon because the nonexistence of a Lehmann representation is sufficient to ensure the absence of gluon production thresholds in $\mathcal{S}$-matrix elements describing colour-singlet to singlet transitions [4,17].

This model, (12), has no explicit $\mu$-dependence, which can arise through quark vacuum polarisation insertions. As such it may be inadequate at large values of $\mu$, particularly near any critical chemical potential. However, until finite-$\mu$ DSE studies of the gluon propagator become available no objective assessment of this possibility can be made. We therefore advocate proceeding with models such as (12) and assessing the qualitative and quantitative results obtained in the light of existing experiments and related theoretical studies.

4

As in Ref. [11] we adopt the additional, simplifying specification:

$$\Gamma_\nu(\tilde{q}, \tilde{p}) = \gamma_\nu \,, \tag{13}$$

which is often referred to as the "rainbow approximation". Using this truncation a mutually consistent constraint is: $Z_1 = Z_2$ and $Z_1^A = Z_2^A$ [16]. In $T = 0 = \mu$ studies this truncation is quantitatively reliable in Landau gauge; i.e., using (13) in (5) to obtain $S(\tilde{p})$ and calculating bound state masses via the Bethe-Salpeter equation, the choice of parameters in $\mathcal{G}(k^2)$ that gives a good description is little modified by requiring the same quality fit with a more sophisticated vertex Ansatz, provided it is free of kinematic, light-cone singularities. Hence we do not expect the explicit form of the vertex to have a significant qualitative effect on our conclusions as long as it is in the class of Ansätze prescribed by Ref. [7]. This is supported by the results of Ref. [6], which indicate that a systematic improvement of (13) has very little effect on flavour-octet, pseudoscalar meson properties.

In the present context, one can apply the confinement test introduced in Ref. [10] by analysing the configuration-space Schwinger function

$$\Delta_S(\tau) \doteq \frac{1}{2\pi} \int_{-\infty}^{\infty} dp_4 \, e^{ip_4\tau} \, \sigma_{B_0}(\vec{p} = 0, \omega_p) \,. \tag{14}$$

(Here and below the subscript "0" denotes a quantity calculated in the chiral limit, $m_R = 0$ [16].)

As an illustration, consider a free, massive fermion, for which $\sigma_B(\vec{p} = 0, \omega_p) = M/(\omega_p^2 + M^2)$. This function has poles at $p_4^2 = -(M \pm \mu)^2$, which are associated with the $\mu$-induced offset of the particle and antiparticle zero-point energies, and one finds $\Delta_S(\tau) = 1/2 \exp[-(M - \mu)\,\tau]\,\theta(M - \mu)$. This function is positive-definite and monotonically decreasing. [1]

In contrast, for a Schwinger function with complex-conjugate $p^2$-poles, $\Delta_S(\tau)$ has zeros at $\tau > 0$ [17]. Such a Schwinger function does not have a Lehmann representation and hence can be interpreted as describing a confined particle, with no associated asymptotic state. The confinement order parameter introduced in Ref. [10] is derived from this observation. Denoting by $\tau_0^z$ the location of the first zero in $\Delta_S(\tau)$, and defining $\kappa \doteq 1/\tau_0^z$, then deconfinement is observed if, for some $\mu = \mu_c$, $\kappa(\mu_c) = 0$; at this point the baryon number density

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[1]  In the context of DSE studies the quantity $M$ can be identified with a constituent-quark mass-scale characteristic of DCSB, which for $u/d$-quarks is $\sim 400\,\mathrm{MeV}$. One therefore expects finite-$\mu$ effects to become noticeable when $\mu \sim 400\,\mathrm{MeV}$.

has overwhelmed the confinement mass-scale and the poles have migrated to the real axis. Obviously, in the case of a free particle $\kappa \equiv 0$.

The simplest order parameter for DCSB is

$$\chi = \Re \left[ B_0(|\vec{p}|^2 = 0, \omega_p^2 = -\mu^2) \right] . \tag{15}$$

Its behaviour at any chiral symmetry restoration transition is identical to that of the chiral-limit vacuum quark condensate.

We solve the DSE for $S(\tilde{p})$, (3), numerically with

$$m_t = 0.69 \, \text{GeV} , \quad m_R(\zeta) = 1.1 \, \text{MeV} , \tag{16}$$

which were fixed in Ref. [11] by requiring a best $\chi^2$-fit to a range of $\pi$-meson observables at $T = 0 = \mu$. We use $d = 4/9$ in (12) and renormalise at $\zeta = 9.47 \, \text{GeV}$. With these choices our present study has no free parameters.

The pion mass is given by [11]

$$m_\pi^2 N_\pi^2 = \langle m_R(\zeta) \, (\bar{q}q)_\zeta \rangle_\pi ; \tag{17}$$

$$\langle m_R(\zeta) \, (\bar{q}q)_\zeta \rangle_\pi = 8 N_c \int_p^\Lambda B_0 \left( \sigma_{B_0} - B_0 \left[ \omega_p^2 \sigma_C^2 + |\vec{p}|^2 \sigma_A^2 + \sigma_B^2 \right] \right) , \tag{18}$$

which vanishes linearly with $m_R(\zeta)$, and

$$N_\pi^2 = 2 N_c \int_p^\Lambda B_0^2 \left\{ \sigma_A^2 - 2 \left[ \omega_p^2 \sigma_C \sigma_C' + |\vec{p}|^2 \sigma_A \sigma_A' + \sigma_B \sigma_B' \right] \right. \tag{19}$$

$$\left. - \tfrac{4}{3} |\vec{p}|^2 \left( \left[ \omega_p^2 \left( \sigma_C \sigma_C'' - (\sigma_C')^2 \right) + |\vec{p}|^2 \left( \sigma_A \sigma_A'' - (\sigma_A')^2 \right) + \sigma_B \sigma_B'' - (\sigma_B')^2 \right] \right) \right\},$$

with $\sigma_B' \equiv \partial \sigma_B(|\vec{p}|^2, \omega_k^2)/\partial |\vec{p}|^2$, etc. $N_\pi$ is the canonical normalisation constant for the Bethe-Salpeter amplitude in ladder approximation. The pion decay constant is obtained from

$$f_\pi N_\pi = 4 N_c \int_p^\Lambda B_0 \left\{ \sigma_A \sigma_B + \tfrac{2}{3} |\vec{p}|^2 \left( \sigma_A' \sigma_B - \sigma_A \sigma_B' \right) \right\} . \tag{20}$$

(17)-(20) were derived [11] under the assumption that the pion Bethe-Salpeter amplitude is given by $\Gamma_\pi = i\gamma_5 B_0$. The limitations of this assumption are discussed in Ref. [16] but it is expected to be qualitatively reliable for small pion
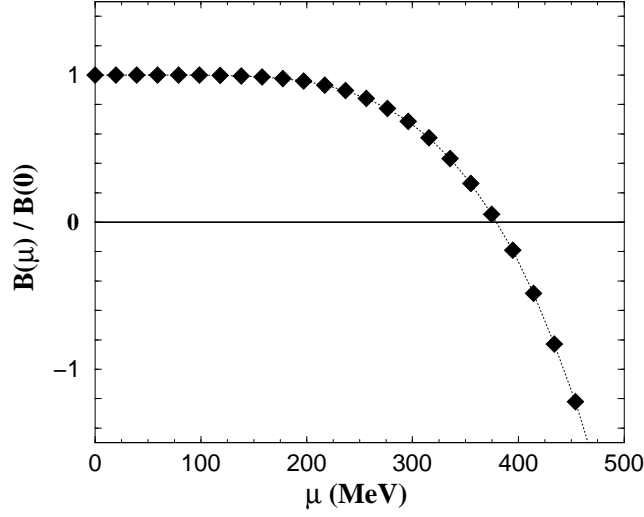
6

Fig. 1. $\mathcal{B}(\mu)$ from (21); $\mathcal{B}(\mu) > 0$ marks the domain of confinement and dynamical chiral symmetry breaking. The zero of $\mathcal{B}(\mu)$ is $\mu_c = 375\,\text{MeV}$. $\mathcal{B}(0) = (0.104\,\text{GeV})^4$.

masses. The difference $\epsilon \doteq |1 - f_\pi/N_\pi|$ is a measure of the error introduced by this representation; and $\epsilon$ is no more than 0.17 over the range of $\mu$ considered. In solving the ladder Bethe-Salpeter equation with $\Gamma_\pi = i\gamma_5 B_0$, (17) provides an estimate of the pion mass obtained thereby that is accurate to within 1%.

**3. Results and Conclusions.** It is now clear that in the domain of confinement and DCSB the solution of the DSE determines the order parameters for deconfinement and chiral symmetry restoration, and elementary pion observables such as $m_\pi$ and $f_\pi$. Since $\mathcal{F} = \mathcal{F}(|\vec{p}|^2, \omega_p^2)$, they are all real.

To explore the possibility of a phase transition we calculate the difference between the tree-level auxiliary-field effective-action [18] evaluated with the Wigner-Weyl solution, characterised by $B_0 \equiv 0$, and the Nambu-Goldstone solution, characterised by $B_0 \not\equiv 0$:

$$\mathcal{B}(\mu) = \tag{21}$$
$$4N_c \int_p^\Lambda \left\{ \ln\left[ \frac{|\vec{p}|^2 A_0^2 + \omega_p^2 C_0^2 + B_0^2}{|\vec{p}|^2 \hat{A}_0^2 + \omega_p^2 \hat{C}_0^2} \right] + |\vec{p}|^2 \left( \sigma_{A_0} - \hat{\sigma}_{A_0} \right) + \omega_p^2 \left( \sigma_{C_0} - \hat{\sigma}_{C_0} \right) \right\},$$

which defines a $\mu$-dependent "bag constant" [19]. In (21), $\hat{A}$ and $\hat{C}$ represent the solution of (3) obtained when $B_0 \equiv 0$; i.e., when dynamical chiral symmetry breaking is absent. This solution exists for all $\mu$.

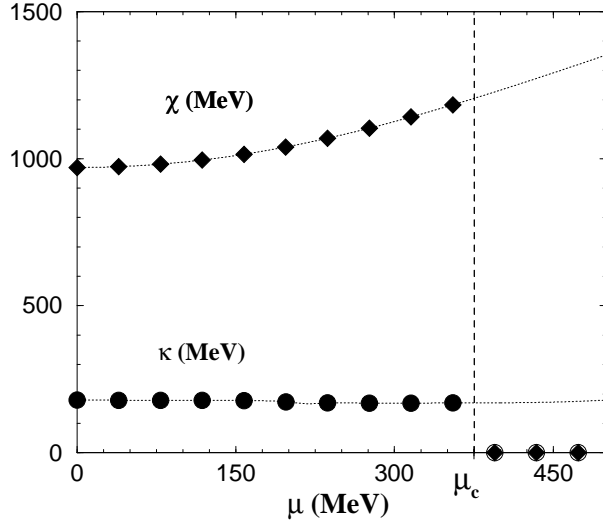$\mathcal{B}(\mu)$ is plotted in Fig. 1. It is positive when the Nambu-Goldstone phase is

7

Fig. 2. The order parameters for chiral symmetry restoration [$\chi$, diamonds] and deconfinement [$\kappa$, circles]. $\mu_c = 375\,\mathrm{MeV}$.

dynamically favoured; i.e., has the highest pressure, and becomes negative when the Wigner pressure becomes larger. The critical chemical potential is the zero of $\mathcal{B}(\mu)$; i.e., $\mu_c = 375\,\mathrm{MeV}$. This abrupt switch from the Nambu-Goldstone to the Wigner-Weyl phase signals a first order transition. The value of $\mu_c$ in Ref. [15], obtained without the second term in (12); i.e., without the "perturbative tail", is $\approx 30\%$ smaller.

In Fig. 2 we plot the order parameters for chiral symmetry restoration, $\chi(\mu)$, and deconfinement, $\kappa(\mu)$, obtained from our DSE solutions. The chiral order parameter *increases* with increasing chemical potential up to $\mu_c$, with $\chi(\mu_c)/\chi(0) \approx 1.2$, whereas $\kappa(\mu)$ is insensitive to increasing $\mu$. At $\mu_c$ they both drop immediately and discontinuously to zero, as expected of a first-order phase transition. The increase of the chiral order parameter with $\mu$ is a necessary consequence of the momentum dependence of the scalar piece of the quark self energy, $B(\tilde{p})$, as is easily seen in Ref. [15] where the qualitative behaviour of both quantities is identical. The vacuum quark condensate behaves in qualitatively the same manner as $\chi$.

The behaviour of $m_\pi$ and $f_\pi$ is illustrated in Fig. 3. One observes that although the chiral order parameter *increases* with $\mu$, $m_\pi$ *decreases* slowly as $\mu$ increases. This slow fall continues until $\mu \approx 0.7\,\mu_c$, when $m_\pi(\mu)/m_\pi(0) \approx 0.94$. At this point $m_\pi$ begins to increase although, for $\mu < \mu_c$, $m_\pi(\mu)$ does not exceed $m_\pi(0)$. This precludes pion condensation, in qualitative agreement with Ref. [20]. The behaviour of $m_\pi$ results from mutually compensating increases in $\langle m_R(\zeta)(\bar{q}q)_\zeta\rangle_\pi$ and $N_\pi^2$. This is a manifestation of the manner in which dynamical chiral symmetry breaking protects pseudoscalar meson
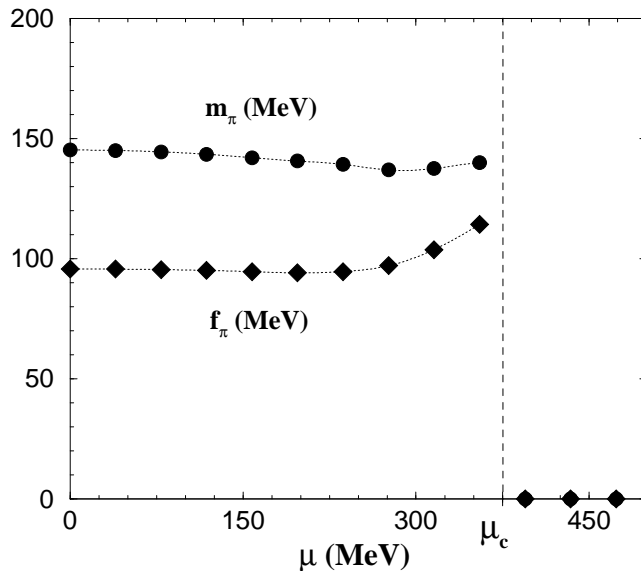
Fig. 3. Chemical potential dependence of the pion mass [$m_\pi$, circles] and pion leptonic decay constant [$f_\pi$, diamonds].

masses against rapid changes with $\mu$. The pion leptonic decay constant is insensitive to the chemical potential until $\mu \approx 0.7\,\mu_c$, when it increases sharply so that $f_\pi(\mu_c^-)/f_\pi(\mu = 0) \approx 1.25$. The relative insensitivity of $m_\pi$ and $f_\pi$ to changes in $\mu$, until very near $\mu_c$, mirrors the behaviour of these observables at finite-$T$ [10]. For example, it leads only to a 14% increase in the $\pi \to \mu\nu$ decay width at $\mu \approx 0.9\,\mu_c$. The conjecture of Ref. [21] is inconsistent with the anticorrelation we observe between the $\mu$-dependence of $f_\pi$ and $m_\pi$.

We expect that improving upon the assumption $\Gamma_\pi = i\gamma_5 B_0$ will only modify these observations to the extent that $f_\pi$ rises slowly and uniformly on $0 < \mu < \mu_c$. The discussion of pion properties has relied implicitly upon the ladder Bethe-Salpeter equation. However, Ref. [6] indicates that improving upon this truncation; i.e., including additional skeleton diagrams in the quark DSE and pion Bethe-Salpeter equation in a manner that preserves Goldstone's theorem, will have little effect on pion properties and hence will not qualitatively affect our conclusions.

The confined-quark vacuum consists of quark-antiquark pairs correlated in a scalar condensate. Increasing $\mu$ increases the scalar density: $\langle \bar{q}q \rangle$. However, as long as $\mu < \mu_c$ there is no excess of particles over antiparticles in the vacuum and hence the baryon number density remains zero; i.e., $\forall \mu < \mu_c$, $\rho_B^{u+d} = 0$ [15]. This is just the statement that quark-antiquark pairs confined in the condensate do not contribute to the baryon number density. After deconfinement the quark pressure increases rapidly, as the condensate "breaks-up", and an excess of quarks over antiquarks develops. At $\mu \sim 5\mu_c$ this quark pressure saturates the Stefan-Boltzmann limit: $P_{u+d} = \mu^4/(2\pi^2)$.

9

As a *gauge* of the magnitude of $\mu_c = 375\,\text{MeV}$ we note that the baryon number density of a two-flavour free-quark gas at this chemical potential would be

$$\rho_B^{u_F + d_F}(\mu_c) = \frac{1}{3}\frac{2\mu_c^3}{\pi^2} = 2.9\,\rho_0 \,, \tag{22}$$

where $\rho_0 = 0.16\,\text{fm}^{-3}$ is the equilibrium density of nuclear matter. For comparison, the central core density expected in a $1.4\,M_\odot$ neutron star is 3.6-4.1 $\rho_0$ [22]. Using this gauge, $\rho_0$ corresponds to $\mu_0 \approx 260\,\text{MeV}$; i.e., $\mu_0 = 0.7\,\mu_c$.

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